Non-Sylvester Hadamard systems such as H_{12}

Non-Sylvester Hadamard systems such as H_{12} introduce qualitatively different behavior compared to the standard power-of-2 Walsh/Sylvester family. The key distinction is that Walsh Hadamards are recursively generated through tensor products, while matrices like H_{12} are globally constrained objects without a natural binary decomposition.

The Sylvester construction is:

H(2n) = [[H(n), H(n)], [H(n), -H(n)]]

This recursive structure embeds:

- binary partition hierarchies,
- Walsh function factorization,
- dyadic frequency structure,
- hypercube symmetry,
- separable coordinate interactions.

As a result, operators such as:

HD + DH

and

HD - DH

where D is a diagonal ±1 matrix, inherit substantial hidden binary organization.

In Walsh systems:

- many rows differ only by low-order parity flips,
- subsets of coordinates repeatedly align,
- eigenspaces often factor into binary subspaces,
- commutators exhibit structured cancellation patterns.

This creates what is effectively a “binary harmonic algebra.”

For a non-Sylvester Hadamard such as H12, these recursive symmetries disappear. The interactions become more globally coupled and less separable.

The symmetric form:

$H_{12}D + DH_{12}$

acts like a globally mixed interference operator rather than a recursively partitioned transform. Coordinate interactions diffuse more uniformly across the system, reducing alignment with binary partitions and dyadic artifacts.

The antisymmetric form:

$H_{12}D - DH_{12}$

is especially interesting because it measures non-commutation between H_{12} and D.

In Walsh systems, many D choices partially commute with portions of the recursive hierarchy. In H_{12} systems:

- accidental commutation is rarer,
- skew interactions spread more globally,
- eigenspaces become less reducible,
- spectral degeneracies weaken.

This likely produces:

- denser coupling graphs,
- more isotropic mixing,
- longer recurrence times,
- fewer binary resonance artifacts,
- more random-matrix-like spectra.

From a graph viewpoint:

- Walsh HD ± DH systems resemble recursively clustered hypercube-like structures,
- $H_{12}D \pm DH_{12}$ resembles a more homogeneous dense interaction graph.

This difference may matter for iterative systems such as:

$x(t+1) = (HD + DH)x(t)$

or normalized variants.

Walsh systems tend to preserve hidden dyadic correlations and can synchronize with:

- parity,
- Gray coding,
- binary counters,
- Walsh harmonics.

Non-Sylvester systems weaken these resonances and may produce more physically generic scrambling dynamics.

Conceptually, the distinction is:

Walsh/Sylvester	**Non-Sylvester**
Binary recursive structure	Global combinatorial structure
Hypercube-like geometry	Dense interference geometry
Bit-factorized modes	Holistic coupled modes
Dyadic harmonics	Non-dyadic spectral structure
Computational flavor	Geometric/interference flavor

Many non-power-of-2 Hadamards arise from:

- finite geometries,
- block designs,
- difference sets,
- Paley constructions,
- quadratic residue structures.

These are mathematically closer to globally balanced interference systems than to recursive Boolean decompositions.

This suggests that $H_{12}D \pm DH_{12}$ systems may behave less like digital logic transforms and more like constrained wave-mixing operators or dense reversible scrambling systems.